

Are non-classical logics revolutionary?

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Lògica: eina i fonament

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Desideratum: non-classical logics have to do with this... *what?*

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[A] system is Euclidean if it is the deductive closure of those of its basic statements [call them 'axioms'] which are assumed to be true. Otherwise it is quasi-empirical.

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- In a Euclidean theory, the true basic statements (the axioms) *prove* the rest of the system; in a quasi-empirical theory the true basic statements are **explained** by the rest of the system.
- In a Euclidean theory, the basic rule is to search for *self-evident axioms*; in a quasi-empirical theory, it is the proliferation of **alternative hypotheses**.

Historiographical consequences of quasi-empiricism

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- The development of Euclidean theory consists of: (1) The naive prescientific stage of trial and error, (2) The foundational period which reorganises the discipline, (3) The solution of problems inside the system.
- The development of a quasi-empirical theory starts with problems followed by solutions, tests and refutations. The vehicle of progress is speculation, criticism, controversy between rival theories and problem-shifts. **The slogans are growth and permanent revolution, not foundations and accumulation of eternal truths.**

Non-classical logics and non-classical mathematics

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Intuitionistic logic

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Relevant logic

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Fuzzy logic

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... and more!

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Is this the *only* role that non-classical logics may have?

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Ifthenism: in order to develop a formal system, the mathematician *only* needs to deal with the formal system itself.

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The problem is that the other signs used in these laws (the universal quantifiers, the equality sign) have been applied in only a finite number of instances, and they can be given non-standard interpretations that will fit non-standard interpretations of '+'.

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A challenge for 'rule-following'.

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A challenge for (non-arbitrary) 'axiom-adding', 'definition-making'.

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*"What you are saying, then, comes to this: a new insight - intuition - is needed at every step to carry out the order '+n' correctly." - To carry it out correctly! How is it **decided** what is the right step to take at any particular point? [...] Or, again, what at any stage we are to call "being in accordance" with it (and with how you then meant it - whatever your meaning it may have consisted in). **It would almost be more correct to say, not that an intuition was needed at every point, but that a new decision was needed at every point.***

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Conclusion: *decision against intuition, grounding against imposition.*

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(2) Mathematics is about the study of mathematical objects *in general*.

(C) Every consistent mathematical theory is worthy of study (leaving pragmatical grounds aside).

Decision *versus* imposition

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The petrification of mathematical activity: *a posteriori* philosophy and persuasion.

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A question of *faith*? Two approaches.

How could this view enable revolution?

Further problems: the normativity of logic

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- Are shifts within logic (as the standard normative system for mathematics) conceivable?
- Do non-classical logics have something to do with these (hypothetical) shifts?
- Could anything like this be found among the history of mathematics?

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